Supplemental WS #5

① LHS $= \sin\theta \cdot \sec\theta$

$= \sin\theta \cdot \frac{1}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$=$ RHS

② LHS $= \cos\theta \cdot \csc\theta$

$= \cos\theta \cdot \frac{1}{\sin\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$=$ RHS

③ LHS $= \frac{\sec\theta}{\csc\theta}$

$= \frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}$

$= \frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta = $ RHS

④ $LHS = 1 - 2\cos^2\theta$

$= 1 - 2(1 - \sin^2\theta)$ ← $\cos^2\theta + \sin^2\theta = 1$

$= 1 - 2 + 2\sin^2\theta$

$= 2\sin^2\theta - 1$

$= RHS$

⑤ $LHS = \cos^2\theta \cdot (\sec^2\theta - 1)$

$= \cos^2\theta \cdot \left(\frac{1}{\cos^2\theta} - 1\right)$

$= \frac{\cos^2\theta}{\cos^2\theta} - \cos^2\theta$

$= 1 - \cos^2\theta$

$= RHS$

⑥ $LHS = (1 + \sin\theta)(1 - \sin\theta)$ FOIL

$= 1 - \sin\theta + \sin\theta - \sin^2\theta$

$= 1 - \sin^2\theta$ ← $\cos^2\theta + \sin^2\theta = 1$

$= \cos^2\theta$

$= \frac{1}{\sec^2\theta}$

$= RHS$

(7) LHS $= \tan^2\theta\,(1-\sin^2\theta)$ $\leftarrow \cos^2\theta+\sin^2\theta=1$

$= \tan^2\theta\,(\cos^2\theta)$

$= \dfrac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta$

$= \sin^2\theta$

$=$ RHS

(8) LHS $= \sin\theta\cdot(\csc\theta-\sin\theta)$

$= \sin\theta\left(\dfrac{1}{\sin\theta}-\sin\theta\right)$

$= \dfrac{\sin\theta}{\sin\theta}-\sin^2\theta$

$= 1-\sin^2\theta$ $\leftarrow \cos^2\theta+\sin^2\theta=1$

$= \cos^2\theta$

$=$ RHS

(9) LHS $= 1-\sin^2\theta\cdot\cot^2\theta$

$= 1-\sin^2\theta\cdot\dfrac{\cos^2\theta}{\sin^2\theta}$

$= 1-\cos^2\theta$

$= \sin^2\theta$

RHS $= \cos^2\theta\cdot\tan^2\theta$

$= \cos^2\theta\cdot\dfrac{\sin^2\theta}{\cos^2\theta}$

$= \sin^2\theta$

LHS $=$ RHS

(12) $LHS = (\tan^2\theta + 1)(1 - \sin^2\theta)$

$= \sec^2\theta \cdot \cos^2\theta$

$= \frac{1}{\cos^2\theta} \cdot \cos^2\theta$

$= 1$

$= RHS$

(11) $LHS = \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta}{\sin\theta \cdot \cos\theta} + \frac{\cos^2\theta}{\sin\theta \cdot \cos\theta}$ ← common denominator

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta}$

$= \frac{1}{\sin\theta \cdot \cos\theta}$

$= \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta}$

$= \csc\theta \cdot \sec\theta$

$= RHS$

(12) LHS $= \dfrac{\sin\theta}{\csc\theta} + \dfrac{\cos\theta}{\sec\theta}$

$= \dfrac{\sin\theta}{\frac{1}{\sin\theta}} + \dfrac{\cos\theta}{\frac{1}{\cos\theta}}$

$= \sin\theta \cdot \dfrac{\sin\theta}{1} + \cos\theta \cdot \dfrac{\cos\theta}{1}$

$= \sin^2\theta + \cos^2\theta$

$= 1$

$=$ RHS

(13) LHS $= \cot\theta + \dfrac{\sin\theta}{1+\cos\theta}$

$= \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta}$

$= \dfrac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)} + \dfrac{\sin\theta(\sin\theta)}{1+\cos\theta(\sin\theta)}$

$= \dfrac{\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \dfrac{\cos\theta + 1}{\sin\theta(1+\cos\theta)}$

$= \dfrac{1}{\sin\theta}$

$= \csc\theta$

$=$ RHS

(14) LHS $= \dfrac{\tan\theta \cdot \sin^2\theta \cdot \cot\theta}{1-\cos\theta}$

$= \dfrac{\frac{\sin\theta}{\cos\theta} \cdot \sin^2\theta \cdot \frac{\cos\theta}{\sin\theta}}{1-\cos\theta}$

$= \dfrac{\sin^2\theta}{1-\cos\theta}$ ← $\cos^2\theta + \sin^2\theta = 1$

$= \dfrac{1-\cos^2\theta}{1-\cos\theta}$ ← dif of squares (factor it)

$= \dfrac{(1+\cos\theta)(1-\cos\theta)}{1-\cos\theta}$

$= 1+\cos\theta$

$=$ RHS

(15) LHS $= \tan\theta \cdot \sin\theta + \cos\theta$

$= \dfrac{\sin\theta}{\cos\theta} \cdot \sin\theta + \cos\theta$

$= \dfrac{\sin^2\theta}{\cos\theta} + \cos\theta$

$= \dfrac{\sin^2\theta}{\cos\theta} + \dfrac{\cos^2\theta}{\cos\theta}$

common denominator

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta}$

$= \sec\theta$

$=$ RHS

(16) LHS $= \dfrac{\cos\theta}{1-\cos\theta} + \dfrac{\cos\theta}{1+\cos\theta}$

$= \dfrac{\cos\theta(1+\cos\theta)}{1-\cos^2\theta} + \dfrac{\cos\theta(1-\cos\theta)}{1-\cos^2\theta}$

$= \dfrac{\cos\theta + \cos^2\theta}{1-\cos^2\theta} + \dfrac{\cos\theta - \cos^2\theta}{1-\cos^2\theta}$

$= \dfrac{\cos\theta + \cancel{\cos^2\theta} + \cos\theta - \cancel{\cos^2\theta}}{1-\cos^2\theta}$

$= \dfrac{2\cos\theta}{1-\cos^2\theta}$

$= \dfrac{2\cos\theta}{\sin^2\theta}$

$= 2 \cdot \dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{1}{\sin\theta}$

$= 2\cot\theta \cdot \csc\theta$

$=$ RHS