

# Supplemental WS #5

$$\begin{aligned}
 \textcircled{1} \quad \text{LHS} &= \sin \theta \cdot \sec \theta \\
 &= \sin \theta \cdot \frac{1}{\cos \theta} \\
 &= \frac{\sin \theta}{\cos \theta} \\
 &= \tan \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad \text{LHS} &= \cos \theta \cdot \csc \theta \\
 &= \cos \theta \cdot \frac{1}{\sin \theta} \\
 &= \frac{\cos \theta}{\sin \theta} \\
 &= \cot \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad \text{LHS} &= \frac{\sec \theta}{\csc \theta} \\
 &= \frac{\frac{1}{\cos \theta}}{\frac{1}{\sin \theta}} \\
 &= \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1} \\
 &= \frac{\sin \theta}{\cos \theta} \\
 &= \tan \theta = \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{4} \quad \text{LHS} &= 1 - 2\cos^2 \theta \\
 &= 1 - 2(1 - \sin^2 \theta) \\
 &= 1 - 2 + 2\sin^2 \theta \\
 &= 2\sin^2 \theta - 1 \\
 &= \text{RHS}
 \end{aligned}$$

$$\leftarrow \cos^2 \theta + \sin^2 \theta = 1$$

$$\begin{aligned}
 \textcircled{5} \quad \text{LHS} &= \cos^2 \theta \cdot (\sec^2 \theta - 1) \\
 &= \cos^2 \theta \cdot \left( \frac{1}{\cos^2 \theta} - 1 \right) \\
 &= \frac{\cos^2 \theta}{\cos^2 \theta} - \cos^2 \theta \\
 &= 1 - \cos^2 \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \quad \text{LHS} &= (1 + \sin \theta)(1 - \sin \theta) \\
 &= 1 - \sin \theta + \sin \theta - \sin^2 \theta \\
 &= 1 - \sin^2 \theta \\
 &= \cos^2 \theta \\
 &= \frac{1}{\sec^2 \theta} \\
 &= \text{RHS}
 \end{aligned}$$

FoIL

$$\leftarrow \cos^2 \theta + \sin^2 \theta = 1$$

$$\begin{aligned}
 \textcircled{7} \quad \text{LHS} &= \tan^2 \theta (1 - \sin^2 \theta) \\
 &= \tan^2 \theta (\cos^2 \theta) \\
 &= \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \cos^2 \theta \\
 &= \sin^2 \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\leftarrow \cos^2 \theta + \sin^2 \theta = 1$$

$$\begin{aligned}
 \textcircled{8} \quad \text{LHS} &= \sin \theta \cdot (\csc \theta - \sin \theta) \\
 &= \sin \theta \left( \frac{1}{\sin \theta} - \sin \theta \right) \\
 &= \frac{\sin \theta}{\sin \theta} - \sin^2 \theta \\
 &= 1 - \sin^2 \theta \\
 &= \cos^2 \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\leftarrow \cos^2 \theta + \sin^2 \theta = 1$$

$$\begin{aligned}
 \textcircled{9} \quad \text{LHS} &= 1 - \sin^2 \theta \cdot \cot^2 \theta & \text{RHS} &= \cos^2 \theta \cdot \tan^2 \theta \\
 &= 1 - \sin^2 \theta \cdot \frac{\cos^2 \theta}{\sin^2 \theta} & &= \cos^2 \theta \cdot \frac{\sin^2 \theta}{\cos^2 \theta} \\
 &= 1 - \cos^2 \theta & &= \sin^2 \theta \\
 &= \sin^2 \theta
 \end{aligned}$$

$$\text{LHS} = \text{RHS}$$

$$\begin{aligned}
 \textcircled{10} \quad \text{LHS} &= (\tan^2 \theta + 1)(1 - \sin^2 \theta) \\
 &= \sec^2 \theta \cdot \cos^2 \theta \\
 &= \frac{1}{\cos^2 \theta} \cdot \cos^2 \theta \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{11} \quad \text{LHS} &= \tan \theta + \csc \theta \\
 &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\
 &= \frac{\sin^2 \theta}{\sin \theta \cdot \cos \theta} + \frac{\cos^2 \theta}{\sin \theta \cdot \cos \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cdot \cos \theta} \\
 &= \frac{1}{\sin \theta \cdot \cos \theta} \\
 &= \frac{1}{\sin \theta} \cdot \frac{1}{\cos \theta} \\
 &= \csc \theta \cdot \sec \theta \\
 &= \text{RHS}
 \end{aligned}$$

} Common denominator

$$\begin{aligned}
 \textcircled{12} \quad \text{LHS} &= \frac{\sin \theta}{\csc \theta} + \frac{\cos \theta}{\sec \theta} \\
 &= \frac{\sin \theta}{\frac{1}{\sin \theta}} + \frac{\cos \theta}{\frac{1}{\cos \theta}} \\
 &= \sin \theta \cdot \frac{\sin \theta}{1} + \cos \theta \cdot \frac{\cos \theta}{1} \\
 &= \sin^2 \theta + \cos^2 \theta \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{13} \quad \text{LHS} &= \cot \theta + \frac{\sin \theta}{1 + \cos \theta} \\
 &= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} \\
 &= \frac{\cos \theta (1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} + \frac{\sin \theta (\sin \theta)}{1 + \cos \theta (\sin \theta)} \\
 &= \frac{\cos \theta + \cos^2 \theta + \sin^2 \theta}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{\cos \theta + 1}{\sin \theta (1 + \cos \theta)} \\
 &= \frac{1}{\sin \theta} \\
 &= \csc \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$(14) \text{ LHS} = \frac{\tan \theta \cdot \sin^2 \theta \cdot \cot \theta}{1 - \cos \theta}$$

$$= \frac{\frac{\sin \theta}{\cos \theta} \cdot \sin^2 \theta \cdot \frac{\cos \theta}{\sin \theta}}{1 - \cos \theta}$$

$$= \frac{\sin^2 \theta}{1 - \cos \theta} \quad \leftarrow \cos^2 \theta + \sin^2 \theta = 1$$

$$= \frac{1 - \cos^2 \theta}{1 - \cos \theta} \quad \leftarrow \text{dif of squares (factor it)}$$

$$= \frac{(1 + \cos \theta)(1 - \cos \theta)}{1 - \cos \theta}$$

$$= 1 + \cos \theta$$

$$= \text{RHS}$$

$$\begin{aligned}
 \textcircled{15} \quad \text{LHS} &= \tan \theta \cdot \sin \theta + \cos \theta \\
 &= \frac{\sin \theta}{\cos \theta} \cdot \sin \theta + \cos \theta \\
 &= \frac{\sin^2 \theta}{\cos \theta} + \cos \theta \\
 &= \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos^2 \theta}{\cos \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} \\
 &= \frac{1}{\cos \theta} \\
 &= \sec \theta \\
 &= \text{RHS}
 \end{aligned}$$

Common denominator

$$\begin{aligned}
(16) \text{ LHS} &= \frac{\cos \theta}{1 - \cos \theta} + \frac{\cos \theta}{1 + \cos \theta} \\
&= \frac{\cos \theta (1 + \cos \theta)}{1 - \cos \theta (1 + \cos \theta)} + \frac{\cos \theta (1 - \cos \theta)}{1 + \cos \theta (1 - \cos \theta)} \\
&= \frac{\cos \theta + \cos^2 \theta}{1 - \cos^2 \theta} + \frac{\cos \theta - \cos^2 \theta}{1 - \cos^2 \theta} \\
&= \frac{\cos \theta + \cancel{\cos^2 \theta} + \cos \theta - \cancel{\cos^2 \theta}}{1 - \cos^2 \theta} \\
&= \frac{2 \cos \theta}{1 - \cos^2 \theta} \\
&= \frac{2 \cos \theta}{\sin^2 \theta} \\
&= 2 \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta} \\
&= 2 \cot \theta \cdot \csc \theta \\
&= \text{RHS}
\end{aligned}$$